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interview with Sharon Griffin

"Doing the Critical Things First"

An aligned approach to preK and early elementary math

Sharon Griffin is an associate professor of education and psychology at Clark University and author of the Number Worlds curriculum for teaching number sense in the preK and elementary years. In this interview with the Harvard Education Letter, *Griffin discusses what cognitive science can teach us about aligning preK and early elementary curriculum and teaching methods with the natural development of children's mathematical thinking.*

What math skills do children need to have before they start school?

Children need to know the counting words ("one, two, three …") and to understand that this reliable sequence of words only gets meaning by being attached to quantities: "We're going shopping and we're going to get *two* boxes of cereal." "You can have *two* cookies but not three." To link these words to the quantities that give them meaning is the crucial thing. Then they can use the counting words by themselves without even needing to see the objects they are counting. That frees children up to do math in their heads. If children establish the link between number and quantity in the prekindergarten years, they will have a solid foundation for future learning.

Many teachers don't teach the link between numbers and quantity. They assume all kids have it.

What kinds of preK experiences do children need to develop these competencies?

To begin with, they need to learn to say the counting words in sequence—what I call the counting string. The crucial thing about the counting words is learning to say them in order. We can't leave anything out, and we can't say anything twice. It's a sequential system.

Along with learning the counting words, kids have to be paying attention to the quantities in their world. The first thing is to be able to describe quantities in terms of polar differences: "This block tower is taller than that block tower." Taller, shorter; heavier, lighter; nearer, farther; hotter, colder. If children are not paying attention to height or weight or distance, then they're not going to be able to map numbers onto those dimensions very readily. In settings where there's rich language, all of that vocabulary mapping onto the world of quantity helps kids understand the quantitative world in a global way. It makes all these quantitative differences salient, so that when they have a grasp of numbers, children can start mathematizing those quantities. All of this happens very naturally in many homes, but most of it is not taught in school (see "Stages in the Development of Number Sense," p. 5).

Children also need to become familiar with all the different ways numbers are represented in our culture. Numbers can be represented as *objects:* Two means two things. Another way is in *patterns*, like the dot patterns on dice or on playing cards. That's already semiabstract, because the quantities are fixed. This makes it harder to grasp that the five pattern is one more than the four pattern—you can't pick one dot up and put it back like you can with objects. Numbers can also refer to *position*. When you're counting in a game like hopscotch, it's not that you're adding one more object but you're moving one more position in space from where you started. A similar representation is *vertical line representation,* like a thermometer, where numbers indicate quantity on a continuous measurement scale. And the fifth way is *dial*

Stages in the Development of Number Sense

Preschool: Counting words and quantities are not well linked
- Recite counting words (one to five or one to ten) in sequence
- Count with one-to-one correspondence
- Make global quantity comparisons (more, less; taller, shorter)

Kindergarten: Counting words and quantities become linked
- Know that small numbers mean a little and big numbers mean a lot
- Know that the next counting word in a sequence means one more
- Use numbers to compare quantities (two fewer cookies, six more steps)
- Can use numbers to make quantity determinations without using objects

First grade: Counting words and quantities are linked to symbols
- Recognize numerals and associate them with counting words
- Grasp the meaning of operational symbols (+, –, =)
- Form "number sentences" to express quantitative relationships (7 – 2 = 5)

Second grade: Children can link words to quantities along two scales (e.g., tens and ones)
- Understand place value
- Tell which two-digit number is bigger
- Mentally solve two-digit addition and subtraction problems
- Solve problems involving hours *and* minutes, dollars *and* cents, weight *and* distance

Third grade: Children consolidate the understandings acquired in second grade
- Apply understandings to broader range of concepts
- Understand multiplication as a relationship among two groups (e.g., three cars with four wheels each)

Source: Adapted from S. Griffin, "The Development of Math Competence in the Preschool and Early School Years," in J.M. Royer, *Mathematical Cognition*.

representations, like a clock, in which quantities increase as you move around the dial clockwise.

We know that many low-income children enter school without the kinds of language-rich experiences that middle-income children have typically had. What other gaps do you see at school entry and how do they affect the development of children's math skills?

More affluent children are exposed to different forms of representing numbers; lower-income kids much less so. Research shows that affluent kids are much more likely to have board games in the home; they may also have dominoes and playing cards. When they play Sorry, for example, they learn that numbers indicate position as they move around the board. To learn that numbers mean position gives you a solid foundation for understanding the ordinal value of numbers.

A lot of kids come into kindergarten with the link between number and quantity firmly established, usually the more affluent kids. A lot of lower-income kids don't get this till they're seven. It's about a two-year delay. And many teachers don't teach the link between numbers and quantity. They assume all kids have it. Even kindergarten teachers now will jump right in and write abstract symbols like "4 + 2" on the board. They're mapping counting words onto numerals and more sophisticated algorithms, and kids don't even have a sense of what the counting words actually mean. By the time these children get to second grade, it's not surprising that many of them hate math! My research shows that children who start kindergarten without this understanding and who do not receive remediation fall farther and farther behind. But with proper, explicit instruction that gives kids exposure to the ways number is represented and allows them to figure out the link between number and quantity, they can catch up with and even surpass their peers.

Once kindergartners have consolidated this understanding, what should come next?

The business of first grade is to integrate the world of counting numbers and the world of quantity with the world of formal symbols. That's when you introduce numerals and plus signs and minus signs and equals signs and link them to the counting words, which children now associate with real quantities. But teachers tend not to do this in the correct sequence. They'll put "5 + 3 = " on the board and tell children, "Use your manipulatives to solve this." But kids may not even know what the symbols mean, and therefore may not have any idea how to represent them with objects. Let's let children solve problems with real quantities—using cubes or weights or steps along a line—let's let them talk about what's happening before they start using numerals and symbols. "How many cubes do you have now? What happened—you got three more? Now how many do you have altogether? How did you figure that out?" Let them talk about adding and subtracting, using the counting words and the language of quantity transactions ("I had five, and I gave three away. Now I have two."). Once that's really solid, *then* introduce the symbols. But don't do it backwards, which is our typical pattern.

This is the reason for the big push for communication in the math classroom. We know that talking makes

Editor's Note

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a huge difference. But it's the one area that teachers are inclined to cut because of time constraints. They want to do a lot of drill and practice to prepare kids for the state accountability tests, so they don't give kids the chance to make sense of the quantity transactions they are enacting, with or without manipulatives. That's a huge tension in elementary schools, and because of that it's more and more important that kids have that rich exposure in preschool and kindergarten, where there's a little more flexibility in what teachers do.

Before they even start school, kids have two to three years of rich experience in the world of language. We need to build on that experience in school instead of forgetting about it, and offer continuing opportunities to link formal written symbols to this rich base of language (see "Developmental Principles for Early Math Instruction," p. 6).

In your ideal world, schools would take a developmentally sequenced approach …

… from preschool right up. Absolutely. Everything builds on that early training. I believe very firmly that the only way you can teach for understanding is to start your instruction from where the child is. If you don't—you can teach kids tricks and rules and they might learn to use them, but they're not going to really understand them. If you don't understand how these fundamental concepts are built, there's no way that you can really interpret where a child is and there's no way you'll have the resources to gear instruction to fit children's needs.

Can you give an example?

At one time or another, almost all children count on their fingers to solve single-digit math problems. To make them embarrassed about doing this or to insist they do the math in their heads deprives them of a wonderful tool for making sense of numbers. Counting on their fingers doesn't hinder cognitive development, it helps it. Children will abandon this tool quite naturally when they no longer need it.

Another example is telling time. Around grade 2, children reach a new stage of cognitive development in which they can start to grasp quantity along two dimensions—for instance, tens and ones, hours and minutes, dollars and cents. At this point it's usually pretty easy for them to learn to tell time. Yet state standards and curriculum guidelines often require students to master this skill in first grade. These requirements aren't appropriate to children's level of understanding and expose them to needless frustration.

You have said that today's curricula are not well aligned with the developmental sequence of children's mathematical thinking.

There are way too many math topics at every level. In the fall of 2006, the National Council of Teachers of Mathematics came up with a new set of curriculum standards called Focal Points, which narrows it down to three areas for each grade level, starting in preK. This is a huge step forward. Still, the amount of time teachers have to teach math is often inadequate. There are many interruptions in the classroom as children come and go for special services, and the children who need quality math instruction the most are often the ones who are pulled out of the classroom for other services.

Many math curricula emphasize "spiraling." But if kids don't get a concept the first time it is taught, they are even less likely to get it the second or third time, when it's usually taught at a higher level. Spiraling prevents the teacher from consolidating knowledge at each level before moving on, which is a basic principle of effective practice. For instance, the function of third grade is to consolidate the knowledge gained from preK through grade 2, in anticipation of the developmental gains that occur around grade 4.

In Asian schools, teachers take the time to teach fewer concepts really well. They'll show students something like a balance beam, they'll let them play with it, they'll let them predict and explain and argue among themselves.

Developmental Principles for Early Math Instruction

Teachers should:

1. Build on children's current knowledge
2. Select learning objectives that are a natural next step for children
3. Make sure children consolidate one level of understanding before moving on to the next
4. Give children opportunities to use number concepts in a broad range of contexts and to learn words for describing quantity in each context (bigger, farther, heavier, hotter)

Activities should:

- Expose children to the major ways numbers are represented and talked about
- Provide opportunities to link quantities, counting words, and symbols
- Provide visual and spatial analogs of number representations for hands-on learning, such as horizontal or vertical number lines children can use to represent and visualize quantity transactions
- Capture children's imaginations so knowledge is embedded not only in their minds but in their hopes, fears, and passions
- Provide opportunities to acquire computational fluency as well as conceptual understanding
- Require the use of metacognitive processes (problem-solving, communication, reasoning) to help children construct knowledge

Source: Adapted from S. Griffin, "Fostering the Development of Whole Number Sense," in M.S. Donovan and J.D. Bransford, *How Students Learn.*

For Further Information

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Number Worlds **http://clarku.edu/numberworlds/**

Then they'll test their predictions and at the end they'll say, "How can we describe this so we'll know what to do next time?" *Then* they start introducing the formal expressions. But we don't have the time. We do it the quick-and-easy way for the kids who understand. We say, "You see this symbol? This is what it means you do."

What do teachers need to know in order to align their math instruction with students' cognitive development?

Preservice teachers come to my classes and they think math is about numbers and rules for manipulating numbers. They don't say, "Math is about quantity" or "Math is a set of conceptual relationships between number and quantity." If you believe that math is about numbers and the rules for manipulating them, of course the best thing you can do is to teach kids the numbers and the rules as early as possible. And when they say "numbers," teachers tend not to think of spoken language. They think of numerals, the written form of numbers. Of course children have to know the rules and the procedures, but they also have to know when it's appropriate to use them. Children need to build that knowledge with rich experiences. The point is not to teach all areas of mathematics from prekindergarten on up, but to cover them gradually over the course of 18 years of schooling. You don't have to do everything in every grade. Let's do the critical things first and do them well. ■

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