Getting On Track Early for School Success:

An Assessment System to Support Effective Instruction

Technical Report:
Creating an Assessment of Math Skills for Three and Four Year Olds

November 22, 2010

Introduction

Over the last five to ten years, supporting young children's mathematical development has become a topic of increasing interest among early childhood educators. In 2002, the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM) published a joint statement on the importance of early childhood mathematics education in which they affirmed that "high-quality, challenging, and accessible mathematics education for 3- to 6-year old children is a vital foundation for future mathematics learning" (NAEYC and NCTM, 2002). A central theme of this joint position paper is that early childhood teachers play a key role in children's long-term mathematical development, just as they play a key role in children's long-term literacy development. The paper similarly communicates to the early childhood community the importance of making preschool classrooms and curriculum "math-rich," in addition to "print-rich" and "language-rich."

The NAEYC/NCTM joint position paper included several general recommendations about children's earliest school experiences with mathematics, but it did not specify exactly what should be taught, or how. Other relatively recent developments have begun to try to address those important questions. For example, early learning standards for mathematics now exist in almost every state. And in 2009, the National Resource Council published a report called

Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity that summarizes the research on children's early mathematics learning and development and describes the implications of this research for teachers and policy-makers. The rising interest in early childhood mathematics is also evidenced by the growing number of preschool mathematics curricula that are currently available, each of which embody beliefs about the mathematics young children should learn, the order in which they should learn it, and the types of experiences that promote that learning.

Even with all of the recent attention and resources, though, it is clear that many early childhood teachers still feel unprepared to provide their students with the "high-quality, challenging, and accessible" mathematics education that the joint NAEYC/NCTM statement called for. Early childhood teachers have gotten the message that they need to be knowledgeable about and attentive to young children's mathematical development, but many of them remain unclear or uncomfortable about what that entails. Part of the problem may be that what teachers (and many others) commonly associate with "school mathematics" is symbolic arithmetic (e.g., 5 + 3 = 8) that is not appropriate for most preschool children. However, the foundational mathematics that *is* appropriate and crucial for preschoolers—such as an understanding of cardinality or the development of spatial concepts and language—has traditionally been developed by children at home, often without the intentional engagement of adults. As a result, early childhood teachers may not be aware of what these foundational mathematical skills, concepts, and understandings are, or what their role as teachers should be in helping children develop them.

We believe that a developmental mathematics assessment can help address this problem by providing a bridge between research and practice. A well-designed assessment tool can efficiently and effectively convey research-based information to teachers about: 1) what the important mathematics content is for young children, and 2) the learning trajectories for each key mathematical content domain. The same tool can also be designed to provide information about children's current skills and understandings within each mathematical content domain, as well as next steps for teaching and learning. Thus, our goals are to develop an objective, valid, and instructionally relevant assessment of children's math skills from ages 3 to 5 that is research-based, useful for teachers, usable by teachers, and developmentally appropriate for children. We believe that developing such a tool—one that is educative for teachers, as well as useful for planning and informing instruction—requires a multifaceted approach that integrates expertise derived from child development research and from practical experience with preschool math instruction, curricula, and assessments. We will first apply these various areas of expertise to review and synthesize the relevant literature on early mathematical development and to review and analyze existing early mathematics assessments. From this foundation, we will use an "engineering" approach, involving multiple cycles of testing and iteration of assessment items in both laboratory and classroom settings, to develop a research-based early mathematics assessment tool that is practical and effective in the classroom (See Figure 1).

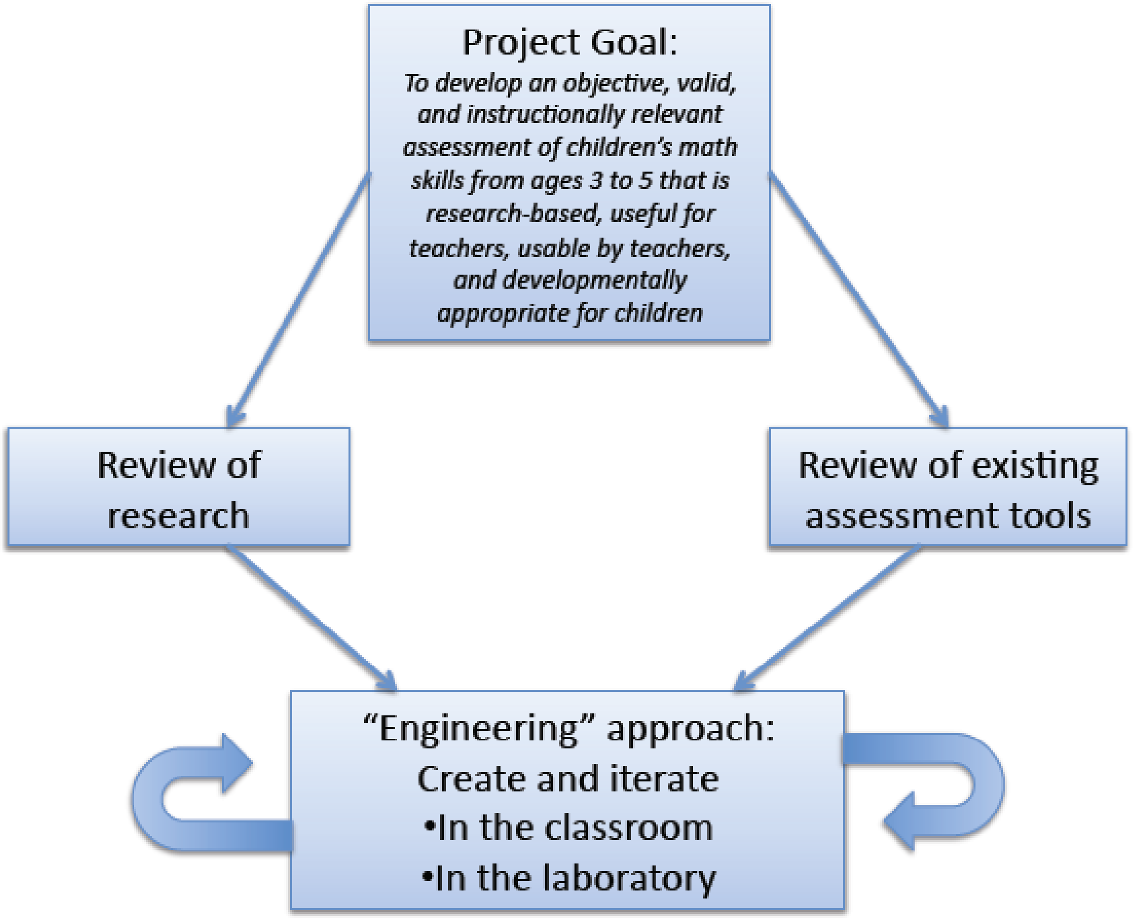

Figure 1. A multifaceted approach to developing an early childhood mathematics assessment.

In this paper, we will focus on the first phase of the work of our project: reviewing the research literature and the existing early childhood assessments. We will draw upon the work of researchers both within and outside of the University of Chicago to present an overview of the relevant research findings in early numeracy and spatial skills, with particular emphasis on early inequalities in math knowledge, foundational spatial and numeracy skills, and the role of language in preschool mathematics. We will also describe the processes we intend to use in conducting a thorough review of existing early childhood mathematics assessments. Finally, we will briefly outline how we will build upon this first phase of work to conduct our second phase of work, in which we will develop and test potential items for an early childhood mathematics assessment through an iterative, engineering approach that involves both laboratory and classroom testing.

Core Domains of Early Mathematical Development

Recognizing an increasing need to improve mathematical literacy beginning in the early years, the Mathematical Sciences Education Board of the Center for Education at the National Research Council established a Committee on Early Childhood Mathematics to review existing research on mathematical development in young children, culminating in a publication which specifies the key mathematical domains that young children learn and develop and developmental trajectories in each identified domain from age two to first grade age (*Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, Center for Education, 2009). This resulting document, informed by NCTM's Curriculum Focal Points, identifies two essential domains for mathematics development that encompass "foundational ideas" that develop in the preschool years: 1) Number, and 2) Geometry, Spatial Thinking and Measurement.

According to this review, the domain of Number (early numeracy) is understood to comprise knowledge about quantity and relative quantity, counting, and representing and interpreting written number symbols, as well as the core relations and addition/subtraction operations through which these concepts are manipulated (see Table 1). Similarly, the core domain of early Geometry, Spatial Thinking and Measurement is organized into sub-components, comprising knowledge about two and three-dimensional objects, spatial relations, and the processes of composition and decomposition. Each of these sub-components may be applied to both spatial thinking and linear measurement tasks, and may be organized into knowledge about parts, knowledge about wholes, and understanding the relationships between parts and wholes (see Table 2 for Spatial Thinking and Table 3 for Linear Measurement).[1]

Developmental trajectories for each of the sub-components of the two core domains are provided with stages of development described as a series of distinct “steps,” with four gradations for Number knowledge (Age 2-3; Age 4; Age 5; Grade 1) and three gradations for Geometry, Spatial Thinking and Measurement (Age 2-3; Age 4; Age 5). The authors acknowledge that the research in this domain is less developed than for number, but that the summary of available literature provides guidance for educators regarding what young children can and should do to develop competence in these areas. The authors of this overview emphasize for both domains that development of components do not take place in isolation, but rather, the use of the various components occur in concert and are inter-related and coordinated within tasks. Furthermore, they note that significant individual differences exist in the timing of skill development, as “a considerable amount of this variability comes from differences in the opportunities to learn these tasks and the opportunity to practice them with occasional feedback to correct errors and extend the learning” (p. 127). Thus, crucial learning and developmental opportunities emerge from the home, child care, preschool and school environments.

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[1] Note: Tables 1, 2, and 3 adapted from: National Research Countil (2009). *Mathematics Learning in EarlyChildhood: Paths Toward Excellence and Equity*. Committee on Early Childhood Mathematics, Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Table 1. Core Components of Number Knowledge in Early Childhood

Cardinality	Providing a number word for the numerosity of a set, such that the last number counted is the number of items in the set	
	Step 1	Uses the process of *perceptual subitizing* (recognizing the number of objects immediately) to give a number for up to 5 things
	Step 2	Understands that numbers 1 to 10 can be composed or decomposed (e.g., 6 = 5 + 1, 10 = 5 + 5)
	Step 3	Recognizes the number 10 in a teen number (e.g., 18 = 10 + 8)
	Step 4	Recognizes tens and ones quantities in numbers 10 to 99 (e.g., 68 = 60+ 8)
Number word list	Using number words to provide a long and ordered list of distinct numbers that have a specific order	
	Step 1	Says 1 to 10
	Step 2	Says 1 to 39
	Step 3	Says the tens list (10, 20, 30…, 90, 100), says 1 to 100 by ones
	Step 4	May count groups of ten using a tens list (1 ten, 2 tens, etc.) as well as the decade list
Counting with one-to-one correspondence	Connecting a number word said in time to an object located in space	
	Step 1	Counts accurately 1 to 6 things
	Step 2	Counts accurately 1 to 15 things in a row
	Step 3	Counts 25 things in a row with effots
	Step 4	Arranges things in groups of ten (or uses prearranged groups or drawings) and counts the groups by tens and then shifts to a count by ones for the leftover single things
Written number symbol	Representing each number in the number list with a unique written symbol	
	Step 1	Knows some symbols, continues to learn new symbols if given such learning opportunities
	Step 2	Reads 1 to 10, writes some numerals
	Step 3	Reads and writes 1 to 19, reads 1 to 100 arranged in groups of ten when counting 1 to 100
	Step 4	Sees that the 0 from the tens number is hiding behind the ones number (e.g., 68 = 60 + 8)
Relations	Understanding concepts of "more than" and "less than"	
	Step 1	Uses perceptual, length, and density strategies to find which is more for two numbers, up to 5
	Step 2	Uses counting and matching strategies to find which is more or less for two numbers, up to 5
	Step 3	Matches or counts to find out which is more and which is less for two numbers, up to 10
	Step 4	Solves comparison word problems that ask, "How many more (less) is one group than another?" up to 18
Operations	Solving addition and subtraction problems	
	Step 1	Uses subitized and counted cardinality to solve situation and oral number word problems up to 5
	Step 2	Uses *conceptual subitizing* (visual composition/decomposition) and

		cardinal counting of objects or fingers to solve situation, word, and oral number word problems up to 8
	Step 3	Uses cardinal counting to solve situation, word, oral number word, and written numeral problems up to 10
	Step 4	Uses *counting on* solution procedures (start counting with the cardinal number of the first addend while keeping track of the second addend) to solve all types of addition and subtraction word problems up to 18

Table 2. Core Components of Spatial Knowledge in Early Childhood

Object Concepts (2D and 3D)	Perceiving, Saying, Describing/Discussing, and Constructing Objects in 2-D Space	
	Step 1	See and describe pictures of objects of all sorts (3-D to 2-D); Discriminate between 2-D and 3-D shapes intuitively, marked by accurate matching or naming
	Step 2	Describe the difference between 2-D and 3-D shapes; Name common 3-D shapes informally and with mathematical names; Identify faces of 3-D objects as 2-D shapes and name those shapes; Informally describe why some blocks "stack well" and others do not
	Step 3	Name common 3-D shapes with mathematical terms (spheres, cylinder, rectangle, prism, pyramid); Begin to use relational language of "right" and "left"; Describe congruent faces and, in context (e.g., block building), parallel faces of blocks
Spatial Relations	Perceiving, Saying, Describing/Discussing, and Constructing Spatial Relations in 2-D Space	
	Step 1	Understand and use relational language, including "in," "out," "on," "off," and "under" along with such vertical directionality terms as "up" and "down"
	Step 2	Match 3-D shapes; Identify (match) the faces of 3-D shapes to (congruent) 2-D shapes; Match faces of congruent 2-D shapes; Name the 2-D shapes
	Step 3	Fill rectangular containers with cubes; Fill one layer at a time; Understand and replicate the perspective of a different viewer
Compositions and Decompositions	Perceiving, Saying, Describing/Discussing, and Constructing Compositions and Decompositions in 2-D Space	
	Step 1	Represent real-world objects with blocks that have a similar shape
	Step 2	Combine building blocks using multiple spatial relations; Compose building blocks to produce composite shapes; Produce arches, enclosures, corners, and crosses systematically
	Step 3	Substitute shapes; build complex structures

Table 3. Core Components of Linear Measurement in Early Childhood

Object Concepts (2D and 3D) and Spatial Relations	Perceiving, Saying, Describing/Discussing, and Constructing Objects and Spatial Relations in 2-D Space	
	Step 1	Informally recognize length as extent of 1-D space; Compare 2 objects directly, noting equality or inequality

	Step 2	Compare the length of two objects by representing them with a third object; Seriate up to six objects by length (e.g., connecting cube towers)
	Step 3	Measure by repeated use of a unit, moving from units that are notably square or cubical to those that more closely embody one dimension (e.g., sticks or stirrers); Seriate any number of objects by length, even if differences between consecutive lengths are not palpable perceptually; Interpret bar graphs to answer questions such as “more,” “less,” as well as simple trends, using length of the bars
Compositions and Decompositions	Perceiving, Saying, Describing/Discussing, and Constructing Compositions and Decompositions n 2-D Space	
	Step 1	Informally combine objects in linear extent
	Step 2	Understand that lengths can be concatenated
	Step 3	Add two lengths to obtain the length of a whole

Individual Differences in Mathematical Development: Early Input Matters

Mathematical development prior to entering school is extremely important for children’s academic success. The math knowledge that children bring to the start of school predicts their math and reading achievement at least through the 5th grade (e.g., Duncan, et al., 2007). However, children show marked individual differences in their mathematical knowledge at the start of school, and these differences are often associated with children’s socioeconomic status (Jordan, Kaplan, Oláh, & Locuniak, 2006; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Lee & Burkam, 2002; Saxe, Guberman, & Gearhart, 1987; Starkey, Klein, & Wakeley, 2004). In preschool and kindergarten, children from lower-SES backgrounds perform less well than children from middle- and higher-SES backgrounds on a variety of math tasks, including recognizing correct counting procedures (Jordan, et al., 2006), ordering the numbers from 1 to 10 (Siegler & Ramani, 2008), and solving simple calculations (e.g., “How much is n pennies and m pennies?", "How much is n pennies take away n pennies?") (Jordan, Huttenlocher, & Levine, 1992; Jordan, Huttenlocher, & Levine, 1994). Differences linked to socioeconomic status are often most pronounced on problems that are more complex and require greater conceptual knowledge, such as understanding cardinality and the reproducing sets, rather than on less complex tasks such as counting or reading Arabic numerals (e.g., Ginsburg & Russell, 1981; Saxe, et al., 1987).

Importantly, these early individual differences in children's math knowledge are related to the mathematically-relevant input that children receive at home and in preschool (Gunderson & Levine, under review; Klibanoff, et al., 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). The amount of talk about numbers that parents provide to their children between 14 and 30 months of age is a significant predictor of children's knowledge of the cardinal meanings of the number words at age 46 months (i.e., knowing that the word "three" refers to sets of three items), even when the parents' SES and overall talkativeness are taken into account (Levine, et al., 2010). Parents' talk about numbers seems to be especially informative when it refers to present object sets (e.g., counting a set of five blocks) as opposed to rote counting (Gunderson & Levine, under review). While it is possible to view these results as a byproduct of the fact that parents and children are genetically related, the same patterns emerge in studies of preschool teachers' math talk (Klibanoff, et al., 2006). The amount of math talk that preschool teachers provide is unrelated to children's math knowledge (knowledge of cardinality, ordinality, calculation, Arabic numerals, shape names, and the concept "half") at the beginning of the year but is significantly related to children's growth in math knowledge over the course of the school year (Klibanoff, et al., 2006). There is also evidence that specific curricula can positively influence children's broad-based math knowledge. For example, in one study of the preschool math curriculum *Pre-K Mathematics*, children in low-income classrooms who used the curriculum did as well by the end of the year as children in middle-income classrooms who did not (Starkey, et al., 2004). These results suggest that enriching children's mathematical lives, both at home and at school, can have a dramatic impact on their early mathematical knowledge.

While many of these studies have focused on early numeracy, spatial skills are a second critical component of young children's mathematical development. Spatial skills are important for success in the STEM disciplines (science, technology, engineering, and mathematics) (e.g., Casey, Pezaris, & Nuttall, 1992; Hegarty, Keehner, Khooshabeh, & Montello, 2009; Wai, Lubinski, & Benbow, 2009). Similarly to early numeracy, individual differences in spatial skills begin to develop by preschool (e.g., Ehrlich, Levine, & Goldin-Meadow, 2006; Levine, Huttenlocher, Taylor, & Langrock, 1999). For example, gender differences in mental rotation ability appear in children as young as 4 years of age, with males, on average, more developed than females (Levine, et al., 1999). In early elementary school, higher-SES children perform better than lower-SES children on mental rotation tasks (Levine, Vasilyeva, Lourenco, Newcombe, & Huttenlocher, 2005). Interestingly, gender differences in mental rotation are present among middle- and high-SES children but not among lower-SES children, suggesting that the gender differences in mental rotation ability may be linked to the types of experiences and activities in which middle- and high-SES children engage (Levine, et al., 2005).

Mathematically-relevant input is as important for the development of spatial skills as it is for numeracy skills. In fact, a recent meta-analysis of spatial training studies has shown that spatial skills can be reliably improved through specific enrichment experiences (Hand, Uttal, Marulis, & Newcombe, 2008). The effect sizes of these training studies are largest for the youngest age groups, highlighting the importance of providing preschool children with experiences that can improve their spatial skills. Children's spatial abilities can be improved through the use of spatial language as well as spatial activities such as puzzle play and block play (Levine, Ratliff, Huttenlocher, & Cannon, under review; Pruden, Levine, & Huttenlocher, under review). Parents' use of spatial language to describe spatial dimensions, properties, and shapes (e.g., “tall”, “curvy”, “circle”) is a significant predictor of children’s own use of spatial language, which in turn predicts children’s performance on nonverbal spatial tasks such as mental rotation and spatial analogies (Pruden, et al., under review). In addition, the amount of time that children engage in puzzles between 2 and 4 years of age, as well as the level of difficulty and children’s engagement in those puzzles, is related to their later mental transformation skill (Levine, et al., under review). Interestingly, although boys and girls play with puzzles with the same frequency, boys play with more difficult puzzles and hear more spatial language during their puzzle play (Levine, et al., under review). Although it is not clear whether the gender difference in children’s puzzle play experiences is driven by parents’ beliefs or by children’s own preferences, this experiential difference likely contributes to the gender differences in children’s mental rotation skill in preschool.

For both numeracy and spatial skills, individual differences in children’s abilities emerge as early as preschool and set children on a higher or lower trajectory for later math achievement. These individual differences are often associated with parents’ SES and, in the case of spatial skills, with the child gender. These early inequities are of great concern, but research on the importance of early input for mathematical skill development gives reason for hope. The more preschool-aged children are given numerical and spatial experiences, the more their skills in these domains improve. In other words, providing developmentally-appropriate math instruction to preschoolers can greatly enhance their knowledge and set them on a trajectory for mathematical success. By helping teachers to effectively monitor and respond to their students’ trajectories of numerical and spatial development, a formative assessment of preschool math can encourage teachers to provide mathematically-relevant experiences in a timely and appropriate manner.

Ongoing Research on Early Spatial Skills

In the domain of early spatial skills, the University of Chicago is at the forefront of research. The University of Chicago is part of the Spatial Intelligence and Learning Center (SILC), an NSF-funded interdisciplinary research center in collaboration with Northwestern University, Temple University, and the University of Pennsylvania. This center is developing a science of spatial learning, and we plan to leverage this cutting-edge knowledge in creating preschool math assessments. One of the major themes of recent spatial research is that children develop spatial reasoning skills at a young age, typically in the absence of any formal instruction. Children as young as preschool age have the ability to solve mental rotation tasks, and some do so using a strategy of visualizing and mentally rotating objects (Estes, 1998). As was mentioned previously, boys as young as 4 years of age outperform girls on mental rotation tasks (Levine, et al., 1999). Recent research has shown that boys and girls differ not only on their overall performance on mental rotation tasks, but also on the strategies they use. Boys tend to gesture about the movement of the pieces more than girls, and the more children gesture, the better they perform (Ehrlich, et al., 2006). In other words, when children use their hands to represent the rotational aspect of the problem, performance improves. This has been confirmed in an experimental training study, in which children were either asked to make a rotation gesture or to point to each piece while solving a mental rotation task (Ehrlich, Tran, Levine, & Goldin-Meadow, 2009). Children who were asked to make the rotation gesture performed significantly better on the mental rotation task than children who were asked to make a pointing gesture (Ehrlich, et al., 2009). Thus, gesture can serve both as an indicator of more advanced strategy use as well as a potential teaching tool.

Mental rotation is the most widely studied mental transformation skill, but other mental transformation skills are also relevant for later STEM achievement. For example, cross-sectioning, which requires inferring a 2D representation of a 3D structure, is a critical skill in biology and geoscience (Cohen & Hegarty, 2007). Previous researchers have argued that the ability to visualize cross-sections does not emerge until the end of elementary school at the earliest (Davis, 1973). However, previous measures of cross-sectioning ability relied on 2D line drawings of 3D objects, which added an additional level of difficulty to the task by requiring children to first translate from 2D to 3D before even attempting to visualize the cross-section. More recent research has sought to reduce these extraneous task demands by showing children real 3D objects or photo-realistic depictions of objects with a plane slicing through (see Figure 2) (Ratliff, McGinnis, & Levine, 2010). In these circumstances, children can succeed at visualizing the cross-sections of 3D shapes at 5 years of age without any training (Ratliff, et al., 2010). Given the great deal of evidence that spatial skills can be improved with training (Hand, et al., 2008), it seems very likely that preschool children can succeed at this type of task given appropriate experiences. Further, cross-sectioning and mental rotation skills were not significantly related once age was controlled, indicating that although both involve mental transformations, they are distinct skills in young children (Ratliff, et al., 2010).

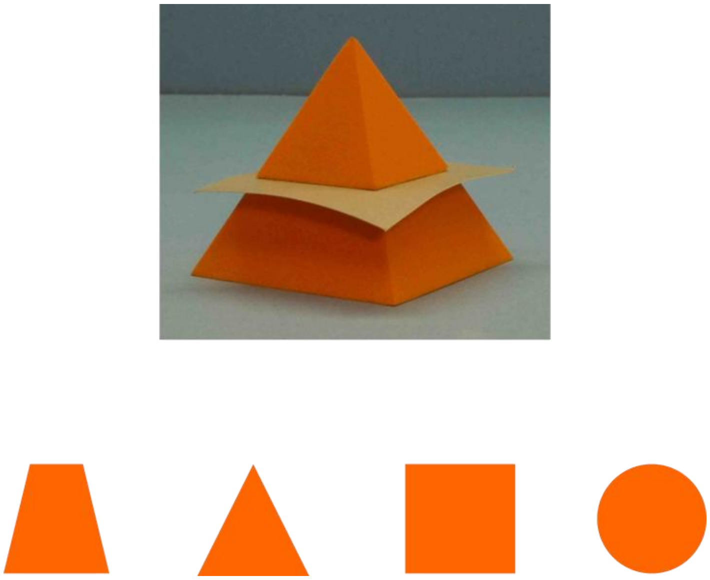

Figure 2. Example of stimulus used in children’s cross-sectioning task (Ratliff, et al., 2010)

Recent research on spatial development sheds light not only on the developmental trajectories of mental transformation skills, but also on effective teaching methods for a more traditional subject in preschool mathematics: shape learning. Because children are often exposed only to prototypical shapes, such as equilateral triangles, their earliest conceptions of shapes are based on their prior experiences with these prototypes (i.e., thinking that a triangle is something that looks like the triangles they’ve seen before) rather than on the defining properties of shapes (i.e., a triangle is a connected figure with three sides and three vertices) (Burger & Shaughnessy, 1986; Clements, Swaminathan, Hannibal, & Sarama, 1999). However, giving preschool children experiences with non-prototypical shapes and providing a guided learning experience where the child discovers the common properties of different exemplars of a particular shape helps children to recognize these defining features at a young age (Fisher, Nash, Hirsh-Pasek, Newcombe, & Golinkoff, April 2009). This research indicates that preschool is not too early for children to begin to learn relatively sophisticated concepts like the defining features of shapes, if these concepts are presented in an engaging and developmentally-appropriate manner.

Ongoing Research on Early Numeracy Skills

In addition to its research on early spatial development, psychologists at the University of Chicago are also heavily involved in research investigating children's early numeracy skills. One of our primary areas of research unpacks the developmental trajectory of learning number words between when children learn to recite the count list (i.e. "one," "two," "three," etc), and when they learn why counting works: that the final number reached when counting a set of objects represents the number of objects in that set (called the "cardinality principle" of counting). It turns out that children do not learn this principle all at once for all numbers. Instead, between about 2½ years and 4½ years, they go through distinct stages of learning single number words. Children first learn that the word "one" refers to sets of one, and will use it correctly, but use all their other number words to mean merely "more than 1." About 9 months later, on average, they learn that "two" refers to a set of two objects. About 6 months after that, they learn that "three" refers to a set of three. These early stages rely on subitizing, a process which allows children (and adults) to accurately determine the numerosity of a small set (up to 3 or 4 items) without counting (e.g., Benoit, Lehalle, & Jouen, 2004; Mandler & Shebo, 1982). Some children also go through a stage where they learn about 4, but around this stage children go through a conceptual shift, learning how all the number words refer to specific set sizes and that counting will help them determine what that number is. At this point, psychologists say they have learned the cardinality principle.

The order of these developmental stages is stable across children and has been replicated in many different labs (e.g., Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Lee, 2009; Wynn, 1992). Moreover, it is a robust phenomenon across tasks and situations. For example, a child who knows meaning of the words “one” and “two” will not only label sets of those sizes consistently correctly, but she will also produce sets of 1 and 2 appropriately when asked for them. However, a child at this stage will simply provide a large number of objects when prompted any quantity more than 2. She will also pick out a set of 1 or 2 objects, when provided two possible options, but when presented with sets of 3 and 4, for example, she will pick a number that varies at random.

While the order of these stages is very stable, children do vary greatly in when and how quickly they go through the stages. In our lab studies that , we 2 ½-year-old children who know the cardinality principle and yet we also see children 4 who have not yet learned the meaning of the word “two.” Our current research suggests that mastery of the cardinality principle is foundational for children’s more global numeracy. Moreover, because learning the meaning of number words is foundational to all higher math, we are also investigating the kinds of instruction that help children, especially those who lag behind, acquire the next concept along this trajectory. Are different kinds of instruction more likely to help children at these different stages of development?

The Role of Language in Early Numeracy

Other early numeracy research at the University of Chicago focuses on the role of language in acquiring numerical concepts. Previous research suggests that language matters for early numerical and spatial skills (Gunderson & Levine, under review; Pruden, et al., under review; Spaepen, 2008). As an extreme example, Spaepen (2008) found that deaf adults who never learned a sign language, and therefore never learned to count in language, also were unable to represent concepts such as “exactly seven,” even nonverbally. Even when children do learn counting from their language, the quality of language input can still affect the rate of acquisition of early numerical concepts. Gunderson & Levine (under review) showed that the kinds of numerical language input children receive from their parents at age 2 affects children's performance on basic number tasks at age 4, providing evidence that early language input can have impacts far later in development.

Beyond the role that language plays in children's acquisition of early number and spatial development, children also show nonverbal evidence of conceptual numerical knowledge before they can demonstrate that knowledge verbally. In particular, children from low-SES backgrounds are sometimes as capable to perform mathematical tasks nonverbally as their high SES counterparts, whereas they will perform significantly worse on tasks tapping the same conceptual knowledge verbally. For example, Jordan, Huttenlocher & Levine (1994) showed that low-income 3 & 4-year-olds succeed on nonverbal responses (i.e. producing a set of disks or pointing to a set of disks) but not verbal responses (i.e. "how many are there?") to nonverbally presented arithmetic problems, whereas no such differences existed for middle- or high-income children.

Moreover, children of all SES backgrounds sometimes exhibit knowledge of numerical concepts nonverbally before they can produce correct answers verbally. For example, research at the University of Chicago and elsewhere shows that children, while going through the stages of number word acquisition described earlier, will sometimes reveal knowledge about the exact number of objects in a set using their fingers before they can use the correct word to identify the set. So, a child who provides a correct verbal answer for sets of 1 and 2 may put up 3 fingers when presented with a set of three objects, but may say "four" or "five" verbally (Spaepen & Le Corre, in prep). Currently, we are conducting research to determine whether those children who display "mismatches" between their verbal and nonverbal responses are most ready to learn the next number word on the developmental trajectory to understanding the cardinal principle.

In sum, the literature and ongoing research on early childhood mathematical development provides evidence that prior to schooling, young children engage in a variety of mathematical experiences across a range of domains of mathematical knowledge, including aspects of both numeracy and spatial skills, that in concert provide the foundations for future mathematics achievement. These findings, which illustrate that young children are capable of demonstrating rich mathematical knowledge, have direct implications for the development of mathematical assessment tools. While our review has focused on research conducted at the University of Chicago in specific domains, we recognize the importance of other domains and plan to conduct a thorough review of research and assessments in those areas.

Evaluating Existing Early Childhood Mathematics Assessments

The existing tools for assessing young children's mathematics knowledge fall into two main categories: 1) assessments used by psychologists and researchers (e.g., *Woodcock-Johnson III Tests of Achievement* (Woodcock, McGrew, & Mather, 2001), *Test of Early Mathematics Ability* (Ginsburg & Baroody, 2003), *Research-Based Early Mathematics Assessment* (Clements, Sarama & Liu, 2008)); and 2) assessments used by teachers (e.g., *Pre-K Everyday Mathematics* Baseline and End-of-Year Assessments (Bell et al., 2008), *Child Assessment Portfolio* (Teaching Strategies, 2010). The assessments used by psychologists and researchers are likely to be grounded in empiricial research and yield detailed descriptions of what children know and can do in a particular mathematical domain or domains. These assessments, however, may have limited utility for teachers due to the time and level of training they often take to administer. In addition, these assessments tend to be diagnostic and descriptive, with relatively less "built-in" attention to instruction or intervention than would be ideal for teacher usage. Many of the mathematics assessments currently used by teachers are curriculum-embedded assessments. These tools have benefits for teachers in that they tend to be directly linked to instruction that has happened or will happen in the classroom. However, they are often too closely linked to a particular curriculum to be broadly useful to a wider group of preschool teachers. They also vary on other key features, such as whether they reflect or convey developmental trajectories, their frequency, their level of specificity, and whether and if so how they are organized into mathematical content domains.

As described previously, an integral part of phase 1 of this project is to thoroughly review the spectrum of existing early childhood mathematics assessment tools, including those used by both teachers and researchers. We plan to conduct a systematic examination of each tool using multiple levels of analysis that focus on item, construct, and instrument-level characteristics. At the item level, we will identify items that are age- and developmentally-appropriate, that are reliable and valid, that predict future math achievement, that measure change in response to instruction, and that meaningfully discriminate between levels of developmental trajectories. We also seek to identify items that are applicable across curricula and are aligned with instructional goals. At the construct level, we expect that this review will provide a better understanding of which domains have been identified and targeted for measurement, which kinds of tasks are used to assess each domain, and ways in which these tools describe and report on student understanding and mis-understanding.

A key question that we will consider as we review the existing assessments relates to the optimal “grain size” for assessment tasks and reporting within each domain that we assess. This is an important question in developing an assessment for classroom usage. For example, the “learning trajectories” outlined by Doug Clements and Julie Sarama as part of their “Building Blocks” curriculum specify 24 levels of development for the skill of counting alone, which is probably too fine-grained to be useful for a teacher who is assessing and planning for 20+

students at a time in all curricular areas. At the other end of the continuum, though, the cc.net assessment system that is associated with Creative Curriculum and widely used in preschool classrooms—including many Head Start programs—only specifies 4 objectives that pertain to mathematics, which is probably not fine-grained enough to help teachers effectively target instruction to promote children's development. (As evidence of this, the recent revamping of the cc.net assessment system actually resulted in a system with a much finer-grained breakdown of skills and understanding within each of the 4 objectives.)

Language demands are another area of focus for our review of existing early childhood mathematics assessments. Previous and current research makes clear that children's language learning is entwined with their learning of early numerical and spatial concepts. English Language Learners (ELL) will certainly struggle to show what they know mathematically if they are only assessed verbally in English. In preschool, though, *all children*, regardless of their native language, are learning the language and vocabulary of mathematics. Therefore, any complete assessment of all preschool children's mathematical knowledge should include supports traditionally used with ELL students. We will include in our review of existing instruments and tasks those with reduced language demands, such as the Number Sense Battery (Jordan, Glutting, & Ramineni, 2008) and nonverbal equivalence tasks (Mix 1999, 2008). Specifically, we will seek assessment items and tasks that incorporate nonverbal assessment strategies to tap conceptual knowledge, such as using concrete manipulatives & demonstrations (e.g., Jordan, et al., 1994), and paying attention to gesture both as a response and a method of communication (e.g., Spaepen & Le Corre, in prep). Moreover, this review of assessments with reduced language demands will inform our understanding of the ways that verbal items may be modified to place as little verbal strain on the children as possible, for example, by using simple sentence frames and by including the correct answer in the questions, such as, “Is this a triangle or a rectangle?” “Is this three or four?”

Next Steps: Applying “Engineering” Principles to Tool Development

Our review of the relevant research literature and the existing early childhood mathematics assessments will provide a foundation for developing an mathematics assessment tool for 3- through 6-year olds that is developmental, independent of curriculum, manageable and practical to use by a teacher in a classroom setting, child-friendly in terms of length and type of task, and educative for teachers both about children’s mathematical development generally and about individual children’s specific mathematical development. The development process will involve creating, testing, and revising items in an iterative fashion, through both laboratory experimentation and piloting in classroom settings. In this “engineering” approach to tool development, individual items will be tested and analyzed for reliability and validity, and groups of items will be piloted with classroom teachers to ensure that the assessment is built upon the best research available and is useful and usable by preschool teachers in a wide range of educational settings.

The ongoing research and iteration during the development process will also be aimed at increasing understanding about the path of developmental trajectories in each of the identified domains, and about the interrelationships between these domains, of which little is known. This ongoing research will also investigate the kinds of instruction that benefit children most at the different stages along this trajectory, study the impact of learning in one domain on knowledge in other domains, and discover how nonverbal knowledge can be used as a diagnostic tool and as an instructional aid when assessing and teaching young children new math concepts.

References

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19*(3), 291-307.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31-48.

Casey, M. B., Pezaris, E., & Nuttall, R. L. (1992). Spatial ability as a predictor of math achievement: The importance of sex and handedness patterns. *Neuropsychologia, 30*(1), 35-45.

Clements, D.H., Sarama, J.H., & Liu, X.H. (2008). Development of a measure of early mathematics achievement using the Rasch model: The research-based early maths assessment. *Educational Psychology, 28*(4), 457–482.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192-212.

Cohen, C. A., & Hegarty, M. (2007). Individual differences in use of external visualisations to perform an internal visualisation task. *Applied Cognitive Psychology, 21*(6), 701-711.

Davis, E. J. (1973). A study of the ability of school pupils to perceive and identify the plane sections of selected solid figures. *Journal for Research in Mathematics Education, 4*(3), 132-140.

Duncan, G. J., Claessens, A., Huston, A. C., Pagani, L. S., Engel, M., Sexton, H., et al. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428-1446.

Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in children's spatial reasoning. *Developmental Psychology, 42*(6), 1259-1268.

Ehrlich, S. B., Tran, K., Levine, S. C., & Goldin-Meadow, S. (2009). Gesture training leads to improvement in children's mental rotation skill. Poster presented at the Biennial Meeting of the Society for Research in Child Development, Denver, CO.

Estes, D. (1998). Young children's awareness of their mental activity: The case of mental rotation. *Child Development, 69*(5), 1345-1360.

Fisher, K., Nash, B., Hirsh-Pasek, K., Newcombe, N. S., & Golinkoff, R. M. (April 2009). *Breaking the mold: Altering preschoolers' concepts of geometric shapes*. Paper presented at the Biennial Meeting of the Society for Research in Child Development.

Ginsburg, H.P. & Baroody, A. (2003). Test of Early Mathematics Ability - Third Edition (TEMA-3). Pro-Ed.

Ginsburg, H. P., & Russell, R. L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development, 46*, 1-72.

Gunderson, E. A., & Levine, S. C. (under review). Some types of parent number talk count more than others: Relations between parents' input and children's number knowledge.

Hand, L. L., Uttal, D. H., Marulis, L., & Newcombe, N. S. (2008). *A meta-analysis of training effects on spatial skills*. Paper presented at the Annual Meeting of the Association for Psychological Science.

Hegarty, M., Keehner, M., Khooshabeh, P., & Montello, D. R. (2009). How spatial abilities enhance, and are enhanced by, dental education. *Learning and Individual Differences, 19*(1), 61-70.

Jordan, N. C., Glutting, J., & Ramineni, C. (2008). A number sense assessment tool for identifying children at risk for mathematical difficulties. In A. Dowker (Ed.), *Mathematical difficulties: Psychology and intervention*. San Diego, CA: Academic Press.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle-income and low-income families. *Developmental Psychology, 28*(4), 644-653.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle- and low-income children. *Learning and Individual Differences, 6*(4), 413-432.

Jordan, N. C., Kaplan, D., Oláh, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*(1), 153-175.

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology*, 59-69.

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology*, 130-169.

Lee, V. E., & Burkam, D. T. (2002). *Inequality at the starting gate: Social background differences in achievement as children begin school*. Washington, D.C.: Economic Policy Institute.

Levine, S. C., Huttenlocher, J., Taylor, A., & Langrock, A. (1999). Early sex differences in spatial skill. *Developmental Psychology, 35*(4), 940-949.

Levine, S. C., Ratliff, K. R., Huttenlocher, J., & Cannon, J. (under review). Early puzzle play: A predictor of preschoolers' mental transformation skill.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology, 46*(5), 1309-1319.

Levine, S. C., Vasilyeva, M., Lourenco, S. F., Newcombe, N. S., & Huttenlocher, J. (2005). Socioeconomic status modifies the sex difference in spatial skill. *Psychological Science, 16*(11), 841-845.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General, 111*(1), 1-22.

Mix, K. S. (1999). Preschoolers' recognition of numerical equivalence: Sequential sets. *Journal of Experimental Child Psychology, 74*(4), 309-332.

Mix, K. S. (2008). Children's equivalence judgments: Crossmapping effects. *Cognitive Development, 23*(1), 191-203.

National Association for the Education of Young Children & National Council of Teachers of Mathematics (2002, 2010). *Early Childhood Mathematics: Promoting Good Beginnings. Joint position statement on math.* Washington, DC: NAEYC.

National Research Countil (2009). Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity. Committee on Early Childhood Mathematics, Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (under review). Children's spatial thinking: Does talk about space matter?

Ratliff, K. R., McGinnis, C., & Levine, S. C. (2010). The development and assessment of cross-sectioning ability in young children. Poster presented at the 32nd Annual Meeting of the Cognitive Science Society, Portland, OR.

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of Experimental Child Psychology*, 325-337.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). *Social processes in early number development*. Chicago: University of Chicago Press.

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science, 11*(5), 655-661.

Spaepen, E. U. (2008). *Language for number: The limits of numerical reasoning without linguistic input.* Unpublished doctoral dissertation, University of Chicago.

Spaepen, E. U., & Le Corre, M. (in prep). Do fingers show earlier evidence of one-to-one correspondence than words?

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, 99-120.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology, 101*, 817-835.

Woodcock, R.W., McGrew, K.S., & Mather, N. (2001). *Woodcock-Johnson III tests of achievement* (Australian ed.). Itasca, IL: Riverside Publishing.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24*, 220-251.